Application of Edge-Elements to 3-D Electromagnetic Field Analysis Accounting for Both Inductive and Capacitive Effects

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***Abstract*—Traditional low-frequency eddy-current solvers do not include displacement current effect while high-frequency solvers do not take into account the nonlinearity of material properties. A novel solver for electromagnetic field analysis that addresses both inductive and capacitive effects is thus highly valuable. A stable time-domain magnetic vector potential (MVP) formulation, which includes both inductive and capacitive effects, is proposed and numerical examples are solved using the proposed formulation. The proposed MVP formulation is very promising in further engineering applications.**

***Index Terms*—Capacitive effect, displacement current, edge-element, finite element method, nonlinear material, three-dimensional.**

I. Introduction

To address the distributed parasitic effects in electromagnetic (EM) devices, such as coupled inductive and capacitive effects in the complicated solid windings in high-frequency transformers or electric machines, it is important to address the displacement current [1-3]. As traditional low-frequency finite element (FE) solvers do not include displacement current while high-frequency solvers do not take into account the nonlinearity in material properties, it is highly desirable to develop a new solver for analyzing these problems.

Edge-elements are commonly and widely used nowadays to approximate vector fields in electromagnetics because of their proper physical sense. Unlike component-wise nodal elements, edge-elements only impose tangential continuity of the fields instead of imposing continuity in all directions. Edge-element is a natural choice for discretizing vector fields because it is more accurate in solving the field discontinuities between the interfaces of air and iron materials [4].

To reduce the number of unknown variables and to make it convenient for considering the nonlinear effects of ferromagnetic materials and coupling with external circuits, the potential formulations [5, 6] are widely used in low frequency eddy-current solvers. There are basically two types of potential formulations, which are, namely, the magnetic scalar potential (MSP) formulations [6, 7] and the magnetic vector potential (MVP) formulations [8-10].

Although the MSP formulations use less degrees of freedom (DoFs), it is a result that is derived from the mathematical aspect rather than from the physical meaning, which is cumbersome in practice. For example, when the current-carrying regions are multiply-connected, which is very common to see, surface cuts or volume cuts have to be introduced [7] to make these regions simply-connected and Ampere's law can then be upheld by assigning suitable jump discontinuities to the MSP or putting the required jump of the MSP to the current vector potential (CVP) [7]. As one can expected, this cut-making process is very complicated and the resultant scalar potential solver is very challenging to develop. As a result, the application of the MSP formulation to problems including both displacement current and eddy-current effects is very difficult and is not a good choice.

In this paper, the edge-elements and their application for problems with both inductive and capacitive effects using the MVP formulation are presented. Those terms with second-order temporal derivative are neglected to make the fully-discrete scheme stable to use. Numerical examples are given to verify the proposed formulation. The stability and feasibility of the developed full-wave Maxwell solver is highly useful when it is necessary to consider coupled inductive and capacitive effects in EM devices.

II. Proposed Time-domain MVP Formulation

A. MVP Formulation

From the differential equations of the full-wave Maxwell system, one can introduce the MVP $\vec{A}$ satisfying $\nabla\times\vec{A}=\vec{B}$; Besides, one can also introduce the electric scalar potential (ESP) φ such that

$$\vec{E}=-\partial\vec{A}/\partial t-\nabla\varphi \ . \tag{1}$$

The MVP ($\vec{A}$ - φ) formulation for the full wave Maxwell problems is stated as

$$\begin{cases}\nabla\times\left(\nu\nabla\times\vec{A}\right)+\sigma\dfrac{\partial\vec{A}}{\partial t}+\sigma\nabla\varphi+\varepsilon\dfrac{\partial}{\partial t}\left(\dfrac{\partial\vec{A}}{\partial t}+\nabla\varphi\right)=\vec{J}_s\\ \nabla\cdot\left(-\sigma(\dfrac{\partial\vec{A}}{\partial t}+\nabla\varphi)-\varepsilon\dfrac{\partial}{\partial t}\left(\dfrac{\partial\vec{A}}{\partial t}+\nabla\varphi\right)\right)=0\end{cases} . \tag{2}$$

In practice, if one directly discretizes the second-order temporal derivative in (2), such as by using the Newmark scheme [11], the resultant scheme becomes unstable. As proposed in [12], one can neglect this term when the effects of wave propagation and radiation are very small. The control equations for problems with both inductive and capacitive effects then become:

$$\begin{cases}\nabla\times\left(\nu\nabla\times\vec{A}\right)+\sigma\dfrac{\partial\vec{A}}{\partial t}+\sigma\nabla\varphi+\varepsilon\dfrac{\partial\left(\nabla\varphi\right)}{\partial t}=\vec{J}_s\\ \nabla\cdot\left(-\sigma(\dfrac{\partial\vec{A}}{\partial t}+\nabla\varphi)-\varepsilon\dfrac{\partial\left(\nabla\varphi\right)}{\partial t}\right)=0\end{cases} , \tag{3}$$

B. Fully-discrete Finite Element Time-Stepping Scheme

For equation system (3), the semi-discrete matrix form can be expressed as (denoted also by $\vec{A}$ and φ, the DoFs associated with the MVP $\vec{A}$ and ESP φ)

$$\begin{cases} \nu K\vec{A} + \sigma M \dfrac{d\vec{A}}{dt} + \sigma K_{AV}\varphi + \varepsilon K_{AV} \dfrac{d\varphi}{dt} = rhs, \\ \sigma K_{VA} \dfrac{d\vec{A}}{dt} + \sigma K_{VV}\varphi + \varepsilon K_{VV} \dfrac{d\varphi}{dt} = 0. \end{cases} \tag{4}$$

The backward-Euler scheme is then used to discretize the first-order temporal derivative.

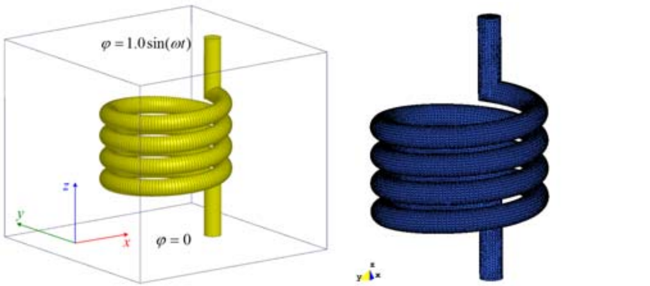
$$\begin{bmatrix} \nu K + \dfrac{\sigma}{\Delta t} M & \sigma K_{AV} + \dfrac{\varepsilon}{\Delta t} K_{AV} \\ \dfrac{\sigma}{\Delta t} K_{VA} & \sigma K_{VV} + \dfrac{\varepsilon}{\Delta t} K_{VV} \end{bmatrix} \begin{bmatrix} A^n \\ \varphi^n \end{bmatrix} = \begin{bmatrix} \dfrac{\sigma}{\Delta t} M & \dfrac{\varepsilon}{\Delta t} K_{AV} \\ \dfrac{\sigma}{\Delta t} K_{VA} & \dfrac{\varepsilon}{\Delta t} K_{VV} \end{bmatrix} \begin{bmatrix} A^{n-1} \\ \varphi^{n-1} \end{bmatrix} . (5)$$

III. Numerical Examples

A. Example 1

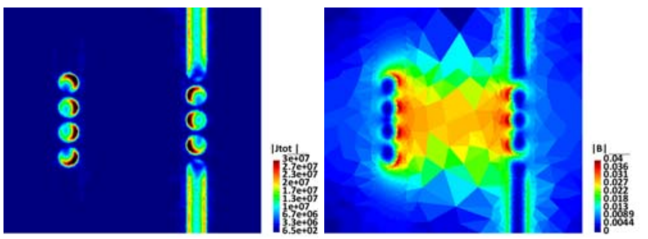
As shown in Fig. 1(a), the helical solid coil is excited by an alternating voltage of $1.0 \cdot \sin(\omega t), \omega = 2\pi f$. The coil is made of copper with an electric conductivity of 5.8×10^7 S/m. The diameter of the coil is 3mm, and the size of the problem domain is $40\text{mm} \times 40\text{mm} \times 35\text{mm}$. The finite element mesh of the helix coil is also shown in Fig. 1(b).

Assuming the frequency *f* of the excitation voltage is 5 kHz and the time-step size is 5μs, the numerical results in the cut plane of *y*=0 after ten steps are given in Figs. 2. In Fig. 2(a), the magnitude of the total current density is given, where both the skin and proximity effects can be clearly observed inside the solid conductor. The magnitude of the vector $\vec{B}$ in the copper coils and the surrounding air is shown in Fig. 2(b).



(a) (b)

Fig.1. (a) The problem domain. (b) The finite element mesh of the solid conductor coil.



(a) (b)

Fig.2. (a) The magnitude of the total current density. (b) The magnitude of the magnetic flux density.

B. Example 2

For this example, a capacitor with alternating voltage excitation is numerically solved in time-domain, as shown in Fig. 3. The radius of the upper terminal of the iron conductor is 1mm; the radius and height of the cylindrical dielectric are 5mm and 0.5mm, respectively. The conductivity of the iron conductor is 10^6S/m.

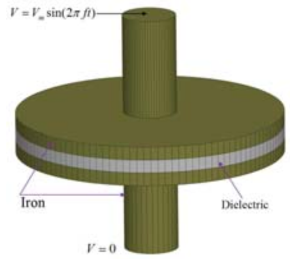


Fig. 3. The capacitor with alternating voltage excitation.

When the excitation peak voltage V_m=1000V, f=10^4Hz and the relative permittivity of the dielectric material is 10^4, the total terminal current versus time-steps is given in Fig. 4.

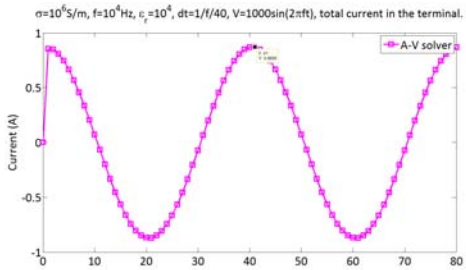


Fig. 4. Current flowing out of the capacitor terminal.

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